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Viscous fingering instabilities in radial Hele-Shaw cell: A review

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ABSTRACT

Viscous fingering (VF) or Saffman–Taylor (ST) instabilities are used in sugar refining, oil recovery, hydrology, filtration, regeneration chemical processing, and tissue engineering. VF arises when a low viscous fluid (μ_1) displaces a high viscous fluid (μ_2) in a Hele-Shaw cell (HSC). Different configurations of HSC have been used for a wide variety of VF pattern formation. These configurations are- radial HSC, lifted HSC and rotational HSC. In this paper, we consider theoretical and experimental works related to VF in radial HSC. VF pattern formation in the case of radial HSC mostly depends on the following parameters - injection rate (Q), gap between the plates (b), viscosity ratio (μ_1/μ_2) and surface tension. A wide variety of miscible and immiscible fluids have been used in the experimental investigation. The ramified fingers pattern appears when displacing and displaced fluids are miscible to each other. This pattern is called a fractal. Interfacial instabilities appear inherently when the viscosity ratio is less than unity.

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1. Introduction

Interfacial instabilities inherently appear only when low viscous fluid is invaded into a high viscous fluid ($\mu_2 > \mu_1$) and invaded fluid displaces the high viscous fluid in Hele-Shaw cell or porous medium. VF patterns form due to unstable displacement of the fluid-fluid interface in the cell. Snowflakes ice formation, dendritic formation in case of solidification of metals, metals deposition in case of electrolytic process and spreading of bacterial colonies are the best examples of VF patterns. The interfacial instabilities are not always undesirable. For example, dendritic formation decreases the life of rechargeable batteries so here undesirable but in case of mixing of two fluids, it is desirable to enhance the mixing efficiency. So that, active control (suppression or enhance) of the VF instabilities are very fascinating. The major active controlling parameters of VF instabilities are - injection rate (Q), gap between plates or fluid film thickness (b), viscosity ratio and surface tension.

The major applications of interfacial instabilities in industrial processes are - CO₂ sequestration [1], enhanced oil recovery [2], water infiltration into the soil, bacterial colonies [3], electrodeposition of metals [4] and so on.

Hele-Shaw cell (HSC) is a device, in which two transparent parallel glass plates are separated by a very small gap (Fig. 1). Interfa-

cial instability in the case of vertical HSC is driven by the combined effect of viscosity force and gravity force (density). But in the case of horizontal HSC interfacial instability is driven by only viscous force. This interfacial instability is used in sugar refining, oil recovery, hydrology, filtration, and tissue engineering. Wide varieties of research have been done on VF pattern formation and observed density and Viscosity play important role in ST instability. But in the case of radial HSC, viscosity is responsible for interfacial instability. Therefore wide varieties of viscous fluids are injected in radial HSC and get a rich variety of fingering patterns [14]. In this paper, we consider theoretical and experimental works related to VF in radial HSC.

2. Experimental setup of radial Hele-Shaw cell

In radial HSC cell (Fig. 1), a stationary thin film (b) of high viscous fluid is presented between two glass plates. One of the plates has a narrow hole at the center. The low viscous fluid is injected by a syringe pump on the surface of thin viscous film through the center hole of the plate. The low viscous fluid flows on the surface of high viscous fluid and pushes it. Then finger-like pattern develops at the fluid-fluid interface. The finger-like path follows by displacing fluid because this path has lower hydraulic resistance. It is called viscous fingering instability and a finger-like structure is called VF pattern.

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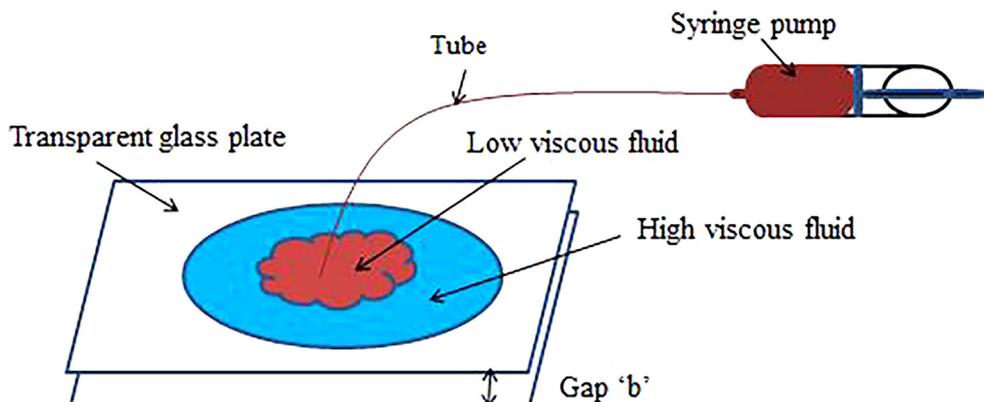


Fig. 1. Schematic diagram of radial Hele-Shaw cell.

3. A wide variety of research works have been done on VF in radial HSC. Few of them are follows

Tao Gao et al. 2019 [5] investigated active control of viscous fingering by applying electric fields. Here active control means that suppressed or enhanced interfacial instabilities in radial HSC by an external source of electric fields. They have been done experiments and observed that viscous fingering form naturally only when viscosities ration less than unity but this instability can also be stabilized by external electric fields. They also found that the viscosity ratio is greater than unity then viscous fingering forms only in the presence of an external electric field. Overall they concluded, interfacial instabilities can be stabilized and destabilized with the help of an electric field. It does not depend on the viscosity ratio when the external electric field present.

Puzovic et al. 2018 [6] have been used thin elastic sheets instead of the top rigid glass plate in radial HSC. The role of the elastic sheet is that it delays the viscous fingering instability when low viscous fluid invades with a large flow rate. Therefore fingers appear in this cell is short and stubby but in case conventional I HSC fingers form in ramified manner. They found, large- scale fingers at a high injection rate. In this study, authors have been selected liquid and air which are immiscible in nature. The air is injected with the content flow rate in the whole experiment.

Yazdi et al. 2018 [7] studied the interfacial instability of two immiscible fluids. The simulation of this study is based on the volume fluid method. They reported, how these dimensionless parameters - mobility ratio, viscosity ratio, mobility factor, elasticity number, and capillary number can be effected the fingering pattern formation. They found, whenever viscosity ratio, elasticity number, and capillary number are increased then sweep efficiency and flow stability increase. The mobility ratio and mobility factor are increased then the sweep efficiency declined.

Niroobakhsh et al. 2017 [8] studied experimentally interfacial instabilities. In this investigation invading and defending fluid have been taken respectively cationic surfactant cetylpyridinium chloride (aqueous surfactant) and oleic acid (fatty acid). Rich varieties of fingering patterns are founded at a different concentration level of surfactant and varying the injection rate. These patterns are different from ST instability. It is seen that when the injection rate is increased then the number of fingers decreases and average finger width increases.

Jackson et al. 2015 [9] examined the impact of dynamic wetting on immiscible fingering pattern formation. This theoretical work is based on Picard scheme. Then dynamic wetting delays the fingers tip splitting and changes the mechanism of splitting from previous. They found finger shielding decreases when the mobility ratio is more and bifurcation delays when the mobility ratio is low.

Bischofberger et al. 2014 [10] investigated the viscous fingering versus stability of two miscible fluids. The miscible fluids develop a ramified fingers pattern. In this study interfacial tension has been taken zero because it prevents short wave fluctuation. They reported a lot of viscous fingering patterns with respect to inner and the outer fluid viscosity ratio. They found, long and highly ramified fingers at high viscosity ratio. Instability is measured by unstable wavelength and unstable wavelength depends upon the viscosity ratio. The first time they introduced the relation between finger length and viscosity ratio. They observed the fingers length decrease when numerical values of viscosity ratio increases.

Anjos et al. 2013 [11] tried to explain the mechanism of interfacial instability and pattern formation in radial HSC. The fluids used in this cell were immiscible in nature. They investigated interfacial instability and tip-splitting mechanism in case of a low capillary number. This theoretical study is based on a mode-coupling approach. In this study, linear and weakly nonlinear dynamics have been applied. The wetting film stabilizes the finger formation and controls finger growth. They found short and stubby fingers in case of wetting film.

Dias et al. 2011 [12] studied the impact of inertia on VF pattern formation. The effect of inertia is that it can be changed the stability and morphology of patterns. They found, whenever the effect of inertia is increased then fingers appear wider and tip splitting increases.

Azaiez et al. 2002 [13] examined the linear interface stability of non-Newtonian fluids. These fluids are miscible in nature. The recilinear HSC has been used. This investigation has been done numerically with the help of the finite difference method. The carreau rheological model is used for the analysis of shear-thinning fluids. This model is based on Darcy's law. They observed that the shear-thinning effects are sufficient to induce dramatic changes in the flow instability. They conclude that shear thinning (non-Newtonian) fluids are less stable than Newtonian fluids.

Petri fasta et al. 2001 [14] investigated the interfacial instability of air and non-Newtonian fluid in the HSC. The primary objective of this study is that to reveals the mechanism behind tip-splitting and dendritic like patterns formation. The air is invaded into PEO solution and air displaces the PEO solution. Due to unstable displacement interfacial instability forms. They provided a parameter which can be suppressed tip splitting.

Ben et al. 1999 [15] investigated the Saffman- Taylor instability and finger behaviors. They used two-dimensional η_c models as well as a power law for viscosity. Authors have been selected non-Newtonian fluid which viscosity depends on the deformation rate. This fluid is confined in the HSC and driving by a fluid which has negligible viscosity. They found finger width tends to zero for extremely more velocity.

Banpurkar et al. 1999 [16] investigated the shape of viscous fingers in the presence of anisotropic in a radial HSC. In this experiment they used miscible fluids (water-glycerin) in an anisotropic radial HSC. They found, the kinetic and surface tension dendrite morphology develop in this investigation independent of driving force.

Poire et al. 1998 [17] purposed a theoretical study on finger pattern formation when the fluid inside the radial HSC is non-Newtonian (shear thinning). The viscosity of this fluid decreases by increasing deformation or local shearing. They used a conformal mapping technique for analysis. They found finger width becomes zero when surface tension goes to zero. The result shows that the finger width drastically decreases in the case of shear thickening fluid.

Sader et al. 1994 [18] examined the effect of non-Newtonian fluids on VF pattern formation. They have done this work theoretically and used power law. In this investigation displacing fluid is Newtonian and displaced fluid is non-Newtonian. Both fluids are mutually immiscible. They found VF patterns appear much earlier in shear-thinning fluid compare to shear thickening fluid.

Chen 1987 [19] studied the viscous fingering patterns in radial HSC experimentally. They used glycerin (miscible) as the defending fluid and water as invading fluid in the one case of experiment and oil (immiscible) is used as invading fluid instead of water in the second case of the experiment. They have seen in this investigation, symmetric dendritic finger patterns can be developed with the help of anisotropy in both cases. In this experiment, they also found in the case of immiscible fluid fingers depend on flow rate. When flow rate increases the finger becomes narrow and side branch increases. But in case of a low flow rate, the fingers become compact. They also investigated plates with smooth surfaces and plates with etched networks on one plate. In this set of experiments, chaotic fingers are developed at the interface. They found the flow rate increases the fingers become narrow and side branches increase. But in case of a low flow rate, the fingers become compact.

Jacob et al. 1985 [20] revealed the role of anisotropy on interfacial pattern formation. They performed experiments on radial HSC and lifted HSC. They used glycerine as defending fluid and air as invading fluid. They created grooving a lattice (engraving a grid) on one of the plates. It is called anisotropy. They formed a rich variety of patterns (faceted growth, tip splitting, needle crystals, and dendrites) in radial HSC with anisotropy. This experiment shows that dendrites like fingers pattern forms in the presence of anisotropy.

4. Summery

We found, following conclusions:-

- Interfacial instabilities form inherently only when the viscosity ratio is less than unity.
- The viscosity ratio is greater than unity then viscous fingering forms only in the presence of external surface forces (electric field).
- The Newtonian fluids form dense branching pattern with respect non-Newtonian.

- The fractal finger generates due to the successive shielding of alternating fingers.
- The injection rate is increased the width of fingers increase but the number of fingers decreases.
- The injection rate is increased the fingers become narrow and side branches increase.
- When a system is highly confined than the fingers amplitude and numbers of fingers are more. The growing fingers are independent of plate confinement.
- The dendrites like fingers pattern can be formed with the help of anisotropy.
- Viscous fingering pattern appears much earlier in shear-thinning fluid compare to shear thickening fluid and shear-thinning fluids are less stable than Newtonian fluids.
- Viscosity ratio, elasticity number, and capillary number are increased then sweep efficiency and flow stability increase and mobility ratio and mobility factor are increased then the sweep efficiency declined.
- Then dynamic wetting delays the fingers tip splitting and finger shielding decrease when the mobility ratio is more.
- Highly ramified and long fingers appear at high viscosity ratio and instability is measured by unstable wavelength.
- The wetting film stabilizes the finger formation and controls finger growth.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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